That the above formula does not give the volume bounded by $f(x, y, z) = 0$, the xy-plane and a cylinder whose elements are parallel to the z-axis, may be readily seen by applying it to the plane $z = c$, in which case it gives a result one-third as large as the correct result.

Also solved by the Proposer.

239 (Number Theory) [March, 1916]. Proposed by HAROLD T. DAVIS, Colorado Springs, Colorado.

Give a general method for determining the solution in integers of the equation

$$x^r - 10xy - (n + 1) + y = 0,$$

where n and r are positive integers.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

Solving for y, $y = \dfrac{x^r - (n + 1)}{10x - 1}$, which must be an integer. Since the denominator is prime to 10 and the numerator integral, y will be an integer if $\dfrac{10^r x^r - (n + 1)\cdot 10^r}{10x - 1}$ is. Dividing algebraically the remainder is $1 - (n + 1)\cdot 10^r$. If then $\dfrac{10^r(n + 1) - 1}{10x - 1}$ is an integer, so is y. Hence the general process (perhaps not that desired) is the following: form $10^r(n + 1) - 1$ and factor it. Equate $10x - 1$ to any factor whose last digit is 9, and we have an integral solution.

261 (Number Theory) [March, 1917]. Proposed by NORMAN ANNING, Chilliwack, B. C.

Show that for any positive integer n (excluding powers of 2) positive integers $a_1, a_2, a_3, \cdots, a_k$ which are less than $n/2$ can be chosen in such a way that

$$2^k \cos (a_1\pi/n) \cos (a_2\pi/n) \cos (a_3\pi/n) \cdots \cos (a_k\pi/n) = 1.$$

SOLUTION BY C. F. GUMMER, Queen's University.

Since n is not a power of 2, it is of the form $(2k + 1)l$. The equation

$$\cos (2k + 1)x - \cos (2k + 1)\alpha = 0$$

has $2k + 1$ distinct roots in $\cos x$, when $\cos (2k + 1)\alpha \neq 1$, the roots being

$$\cos \left(\alpha + \frac{2i\pi}{2k + 1} \right), \qquad i = 0, 1, \cdots, 2k.$$

Also $\cos (2k + 1)x = 2^{2k} \cos^{2k+1} x - \cdots$, the absolute term being zero. Hence,

$$2^{2k} \prod_{i=0}^{2k} \cos \left(\alpha + \frac{2i\pi}{2k + 1} \right) = \cos (2k + 1)\alpha.$$

By taking the limit of each side when $\alpha \rightarrow 0$,

$$2^{2k} \prod_{i=0}^{2k} \cos \frac{2i\pi}{2k + 1} = 1,$$

that is,

$$\left\{ 2^k \prod_{i=0}^{k} \cos \frac{2i\pi}{2k + 1} \right\}^2 = 1,$$

or

$$2^k \prod_{i=0}^{k} \cos \frac{2i\pi}{2k + 1} = \pm 1.$$

By taking $j = 2i$, when $i \leqq k/2$ and $j = 2k + 1 - 2i$ when $i > k/2$, we get

$$2^k \prod_{j=1}^{k} \cos \frac{j\pi}{2k + 1} = + 1,$$

which takes the required form if $a_j = jl$.